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Moving Observers in an Isotropic Universe

Subhash Kak

Abstract

We show how the anisotropy resulting from the motion of an observer in an isotropic universe may be determined by measurements. This provides a means to identify inertial frames, yielding a simple resolution to the twins paradox of relativity theory. We propose that isotropy is a requirement for a frame to be inertial; this makes it possible to relate motion to the large scale structure of the universe.

1 Introduction

The twins (or clock) paradox presents a central problem related to the interpretation and use of the relativity principle, namely how to identify inertial frames. The problem concerns two twins, one of whom leaves the earth on a spacecraft moving with a speed comparable to the speed of light. After quick acceleration, the craft moves in uniform motion to a distant star, swings around and returns to the earth. When the traveling twin has returned home, he finds he is younger to the twin who remained earth-bound even though with respect to him, it is the earth-bound twin who had been in motion.

Excepting for brief intervals when the traveling twin was accelerating, the twins, according to the relativity theory, belong to inertial frames and, therefore, the situation appears perfectly symmetrical.

There exist many different "resolutions" to the paradox. The most common of these invokes asymmetry as the twin who leaves the earth undergoes acceleration whereas the earthbound twin does not. Another explanation is based on the asymmetry in the Doppler-shifting of light pulses received by the twins from each other in the outward and inward journeys. Yet another explains the age difference to the switching of the inertial frames by the traveling twin. These various "resolutions" are not in consonance with each other.

The slowing down of all clocks and processes – including atomic vibrations – on the traveling twin cannot be laid on the periods of acceleration and turning around during the journey, since they can, in principle, be made as small as one desires. Furthermore, if there is a slowing down of the clock of the traveling twin on the complete trip, a component of this slowing down must have occurred on the outbound trip itself, making the paradox even more acute.

Einstein's own "resolution" in 1918, which was an attempt to counter the criticism related to the paradox until that time [1], used the gravitational time dilation of the theory of general relativity to explain the asymmetrical time dilation of the traveling twin. This explanation is generally considered wrong (see e.g. [2]), and is different from the other resolutions recounted earlier.

The diversity and the mutual inconsistency of the offered solutions only reinforces the reality of the paradox within relativity. According to the recent assessment by Unnikrishnan [2], " The failure of the accepted views and resolutions can be traced to the fact that the special relativity principle formulated originally for physics in empty space is not valid in the matter-filled universe."

In this article, we present a new principle for the identification of inertial frames in a matter-filled universe that allows us to easily resolve the twins paradox. The principle implies that the identification of a frame as being inertial depends on whether the universe has spatial isotropy with respect to it. This is equivalent to determining the motion of objects against the background of distant stars.

2 Laws and the nature of the universe

According to the principle of relativity, systems of reference moving uniformly and rectilinearly with respect to each other have the same laws, and the speed of light is constant in all such systems. "Laws" are defined in operational terms, by means of readings on instruments that are to be used in a clearly specified manner in all inertial reference frames, in which free particles move in straight lines.

Poincaré enunciated the principle of relativity in 1904 in following words [3], [4]:

1. The laws of physical phenomena must be the same for a 'fixed' observer as for an observer who has a uniform motion of translation relative to him: so that we have not, and cannot possibly have, any means of discerning whether we are, or are not, carried along in such a motion.

2. From all these results there must arise an entirely new kind of dynamics, *which will be characterised above all by the rule, that no velocity can exceed the velocity of light.*

Neither Poincaré nor Einstein, with his similar statement of the relativity principle [5], considered its implications for determining the physical nature of the universe.

Before the advent of the principle of relativity, it was popular to conceive of the universe as being suspended in absolute space. Given the unstated assumption of a finite universe, it led to the notion that the centre of gravity of the universe should be considered to be absolutely at rest, and the plane in which the angular momentum of the universe around this centre is the greatest, should also be considered to be absolutely at rest.

In the late 19th century, the success of the wave theory of light spurred scientists to determine the earth's motion relative to the aether that was taken to be the medium in which the waves propagated. But the failure of the efforts to measure this velocity led Poincaré in 1899 to declare that "absolute motion is indetectible, whether by dynamical, optical, or electrical means." [6]

It seems reasonable to assume that physical processes are a consequence of the large scale nature of the universe as is clear from the Coriolis forces that tell us that the earth is rotating with respect to distant stars. Likewise, the earth's rotation is inferred by the fact that it is slightly flattened at the poles due to the centrifugal forces.

Since the imperative in science is to take the laws to be the same everywhere, the universe must be isotropic. Expressed differently, the universal application of the principle of relativity is a consequence of the fact that the world is isotropic.

By extension, if the universe deviated from isotropy and if its structure was different in the past, the constants of physical laws (or perhaps the laws themselves) will vary with respect to location and time. Indeed, there are current theories that propose variation of speed of light as well as variation in the fine structure constant in the past.

If the laws are independent of the size and the structure of the visible universe then this universe may be infinite in extent. In such a case, all inertial frames in mutual uniform motion must be equivalent, as supposed by Poincaré and Einstein.

3 Distribution of speeds

Consider an isotropic universe in which objects are receding from the observer with speeds that vary uniformly over $(-1, 1)$, where the speed of light, c, is taken to be 1. The observer can make measurements of Doppler shifts and conclude that the probability density function of the speeds, x, is uniform:

$$f_X(x) = \frac{1}{2}, \quad -1 \leq x \leq 1 \tag{1}$$

Now, suppose the observer starts moving in a specific direction with speed v. The probability density function of the speeds of the objects with respect to the observer would be changed.

The observer can measure the Doppler with respect to the distant receding objects in the antipodal directions related to the motion. Let the new variable of speed be y. By the law of composition of velocities in relativity theory, the new speed will be given by the equation:

$$y = \frac{x + v}{1 + xv} \tag{2}$$

Since the mapping between x and y is a monotonic function, one can determine the probability density function of the variable y by a simple transformation rule on random variables. First, we compute the derivative of the mapping between x and y:

$$\frac{dy}{dx} = \frac{1 - v^2}{(1 + xv)^2} \tag{3}$$

This implies that the new probability density function is:

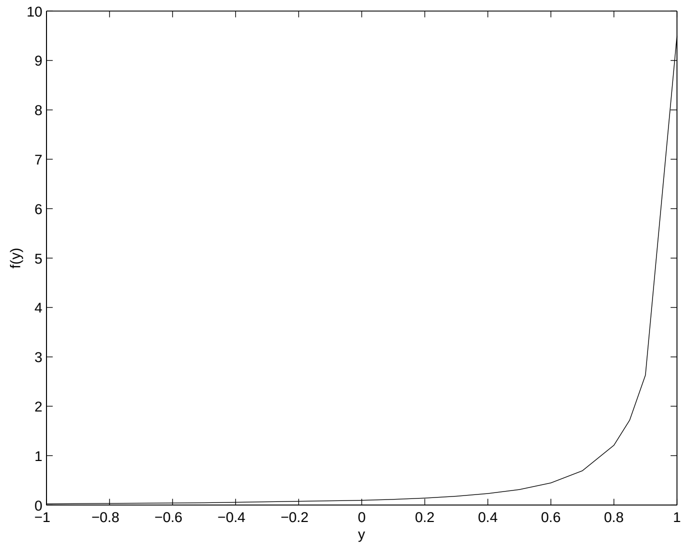

Figure 1: Probability density function of new speeds for v=0.9

$$f_Y(y) = f_X(x)\left|\frac{dx}{dy}\right| = \frac{1}{2}\frac{(1+xv)^2}{(1-v^2)} \tag{4}$$

Written in terms of the variables y and v alone, we have

$$f_Y(y) = \frac{1}{2}\frac{(1-v^2)}{(1-yv)^2}, \quad -1 \leq y \leq 1 \tag{5}$$

Thus the speed of the observer may be inferred in principle by measurements of the distribution of speeds of the receding distant objects in the direction of the motion.

Example. If $v = 0.9$, that is if the observer moves with a speed that is 90 percent of the speed of light, the distribution of the recessional speeds of the distant stars in the direction of the motion will be given by

$$= \frac{1}{2}\frac{0.19}{(1-0.9y)^2}, \quad -1 \leq y \leq 1 \tag{6}$$

Figure 1 presents the probability distribution function for y for this case of $v = 0.9$. The anisotropy in the directions of motion is great. To such an observer, more distant objects in the direction of the motion would appear to recede at speeds that are close to the speed of light. Furthermore, there is marked anisotropy with respect to the antipodal direction.

To see the values at the extreme points, we first consider $y = 1$.

$$f(y = 1) = \frac{1}{2}\frac{(1 + v)}{(1 - v)} \tag{7}$$

At the other end, when $y = -1$,

$$f(y = -1) = \frac{1}{2}\frac{(1 - v)}{(1 + v)} \tag{8}$$

In other words, the anisotropy is determined most clearly by considering objects that are furthest to the observer.

In principle, it is not essential for the speeds distribution to have been uniform in the beginning to estimate a departure from it.

4 Anisotropy and flow of time

Consider an observer, A, who is at rest at origin O of an inertial frame. Observer, B, is also at rest with respect to the same frame. Now B quickly accelerates and then moves off at uniform velocity and after reaching a point C, he reverses his direction and returns to O with the same velocity. Both observers carry ideal clocks. The clocks of A and B will show local time as T_A and T_B, where

$$T_A = \frac{T_B}{\sqrt{(1 - \frac{v^2}{c^2})}} \tag{9}$$

If $v \ll c$, one can write this equation as

$$T_A \approx T_B(1 + \frac{v^2}{2c^2}) \tag{10}$$

In other words, if A and B are twins, and $v \ll c$, then twin B would be younger roughly by the amount:

$$T_A - T_B \approx \frac{v^2}{2c^2} \times T_B \tag{11}$$

Because of the symmetry of the travel, a slowing down must occur in both the outbound and the inbound legs of the journey. Therefore, the slowing down must be the consequence of a real difference between the two frames with respect to the rest of the universe.

We propose that the difference in aging is a function, g, of the two probability density functions, where we now use the same index set x. Or,

$$T_A - T_B = g(f_X(x), f_Y(x)) \tag{12}$$

If the difference between the two times is a logarithmic function of the ratios of the probability density functions, it ensures that the difference is zero for two frames that have no relative motion:

$$T_A - T_B = g(\log(f_X(x)/f_Y(x))) \tag{13}$$

One might also use a relative entropy measure to compare the two distributions:

$$H(X:Y) = \sum f_X(x) \log \frac{f_X(x)}{f_Y(x)} \tag{14}$$

Having seen that the uniform motion with respect to the distant stars is detectable, it is possible to determine which of the two twins is to be taken to be inertial. This means that the slowing down of clocks is a consequence of the universe not being perfectly isotropic to it.

Our analysis shows that Poincaré's and Einstein's implicit equivalence of all frames in uniform motion in the principle of relativity is incorrect in a finite universe.

Time dilation is seen as a consequence of Lorentz transformations, but it may also be viewed as a consequence of the anisotropic apprehension of the universe by the moving observer. This allows easy resolution of various situations related to observers.

For example, if two travelers leave in arbitrary directions with the same speed and return to an inertial frame, their own clocks would still be synchronized, but lag behind that of the inertial frame.

One could also speak of a set of frames that can be put in an hierarchical relationship in terms of their suitability as being inertial.

5 Concluding remarks

If it is assumed that "physical laws" are a consequence of the large scale nature of the universe, then it is valid to assume that there will be a difference in the experience of two observers in relative uniform motion if isotropy of the universe is not maintained by them equally.

Could one say that the assumption of a finite universe goes against the relativity principle as normally defined because in such a universe it is possible, in principle, to calculate the distribution of speeds? If the universe is infinite in size then the change in its isotropy with respect to the moving observer may not materially alter the isotropy maintained with the non-visible part of the universe. In such a case, Poincaré's and Einstein's claims regarding the equivalence of all frames in relative uniform motion remain valid, but at the cost of the twins paradox.

It must be stressed that the determination that an observer is in motion with respect to distant stars does not imply a return to absolute space. An isotropic universe will have an infinity of inertial frames.

References

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6. H. Poincaré, quoted in Whittaker [3], page 30.